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Sets with atoms (also known as permutation models, or Fraenkel-Mostowski sets) were introduced in set theory by Fraenkel in 1922, and further developed by Mostowski. In the last decade, they were rediscovered for the computer science community, where they are called *nominal sets*. Nominal sets are an active research topic in the semantics of programming languages, see the book [3].

This talk is about a research programme, which studies a notion of finiteness which only makes sense in sets with atoms, called "orbit-finiteness". The research programme is to see what happens to discrete mathematics when sets are replaced by sets with atoms, and finiteness is replaces by orbit-finiteness. Two examples of this research programme, motivated by computer science, are orbit-finite versions of programming languages, and of Turing machines.

- **Programming languages.** In sets with atoms, the set of rational numbers with their order is an orbit-finite set. Therefore, in a programming language for sets with atoms, one can write the following program, which exhaustively searches through all triples of rational numbers, searching for a counter-example to transitivity of the order:

```
counterexample:=false
for x in Q do
  for y in Q do
    for z in Q do
       if  (x < y) and (y < z) and not (x < z) then
          counterexample:=true
```

  This programme will be executed in finite time, and will never set `counterexample` to `true`. The programming language is described in [1].
- **Turing machines.** When Turing machines with orbit-finite state spaces and alphabets are considered in [2], the computability landscape becomes richer, in the sense that there are fewer equivalences between models. For instance, P is provably different from NP (although this result is unlikely to shed any light on the real P vs NP problem). Also, nondeterministic decidability does not imply deterministic decidability (unlike for normal Turing machines, where the two notions coincide).

[1] Mikołaj Bojańczyk, Szymon Toruńczyk, *Imperative Programming in Sets with Atoms*, ***Proceedings of the IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science***, 2012, pp. 4–15.

[2] Mikołaj Bojańczyk, Bartosz Klin, Sławomir Lasota, Szymon Toruńczyk, *Turing Machines with Atoms*, ***Proceedings of the 28th Annual IEEE Symposium on Logic in Computer Science***, 2013, to appear.

[3] Andrew M. Pitts, ***Nominal Sets: Names and Symmetry in Computer Science***, Cambridge Tracts in Theoretical Computer Science, Cambridge University Press, 2013.