Inner Models Constructed From Generalized Logics

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outline

Joint work with J. Kennedy and J. Vaananen (A work in progress)

The search for canonical inner models is an attempt to deal with incompleteness

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2. Completeness: Canonical definable objects should be included.
   Litmus test: Closure under sharps or other canonical operations.

Universe constructed from Generalized Logic

Generalized Logic $\mathcal{L}$ has two components (S, T) where S is the set of formulas (which may have free variables) and T is the truth predicate relation, between a model M, a formula Φ and an assignment to the free variables $\vec{a}$

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Definition

For a logic $\mathcal{L}$ and a set M we denote by $Def_{\mathcal{L}}(M)$ the collection of all subsets of M definable in the logic $\mathcal{L}$ in the structure $< M, \varepsilon >$ in the logic $\mathcal{L}$ using parameters from M.

Inner constructed by the Logic $\mathcal{L}$

Definition

Given the logic $\mathcal{L}$. The sequence of sets $L_\alpha^\mathcal{L}$ is defined by induction on the ordinal α:

1. $L_0^\mathcal{L} = \emptyset$
2. For α limit $L_\alpha^\mathcal{L} = \bigcup_{\beta<\alpha} L_\beta^\mathcal{L}$
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The inner model constructed by the logic $\mathcal{L}$ is $\mathcal{C}(\mathcal{L}) = \bigcup_{\alpha\in On} L_\alpha^{\mathcal{L}}$

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The class of hereditarily ordinal definable sets (a.k.a. HOD) is exactly $\mathcal{C}(\mathcal{L})$ where $\mathcal{L}$ is second order logic.

HOD has maximal completeness , canonical objects are ordinal definable. It is somewhat robust under changes in the definition, but non robust across universes of Set Theory.

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Theorem (Kennedy-M.-Vaananen)

There is a model of Set Theory in which $\mathcal{C}(\Sigma^1_1) \neq HOD$ where $\mathcal{C}(\Sigma^1_1)$ is the model constructed by the logic which contains only Σ^1_1 formulas.

Some extensions of first order logic

1. $\mathcal{L}(Q^{WF})$ is first order logic with the additional quantifier ("The well foundedness quantifier") Q^{WF} where $Q^{WF}xy\Phi(x,y)$ means " $\Phi(x,y)$ defines a binary relation that is well founded".

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1. $\mathcal{L}(Q^H)$ is first order logic with the additional quantifier ("The Härtig quantifier") where $Q^H xy\Phi(x)\Psi(y)$ means that the two sets $\{x|\Phi(x)\}$ and $\{y|\Psi(y)\}$ have the same cardinality.

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2. $\mathcal{L}(Q^{aa})$ is the logic ("stationary logic") is first order logic with the additional quantifier Q^{aa} where $Q^{aa}X\Phi(X)$ (X is a second order variable) meaning in a model M $\{X \in P_{\omega_1}(M)|\Phi(X)\}$ is a stationary subset of $P_{\omega_1}(M)$. ($P_{\omega_1}(M)$ is the collection of countable subsets of M .)

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$\mathcal{C}(Q_1^{MM})$ can be changed by forcing over L.
We can define in L a sequence $\langle T_\alpha | \alpha < \omega_2 \rangle$ of Souslin trees on ω_1 which are independent in the sense that we can destroy the Souslinity of some without destroying the Soulinity of others. Using that We can code a non constructible subset of ω_2 as the set $B = \{\alpha < \omega_2 | T_\alpha \text{is Soulin}\}$. Since one can express in $\mathcal{L}(Q_1^{MM})$ that $(T, \prec)$ is a Souslin tree then one gets $B \in \mathcal{C}(Q_1^{MM})$. So we can have models in which $\mathcal{C}(Q_1^{MM} \models 2^{\aleph_0} = \aleph_1$ as well as models in which $\mathcal{C}(Q_1^{MM} \models 2^{\aleph_0} = \aleph_2$.

The effect of $0^\sharp$ on $\mathcal{C}(Q_1^{MM})$

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Sketch of proof: The main lemma is:

Lemma

Assume $0^\sharp$ exists . Let A be a subset of unordered pairs such that $A \in L$. Then there is a set B such that $|B| \geq \omega_1$ and $[B]^2 \subseteq A$ iff $L \models \exists B(|B| \geq \omega_1 \wedge [B]^2 \subseteq A)$

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Theorem

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The proof is based on the following lemma:

Lemma

X is a set of ordinals such that $X^{\sharp}$ exists. Let λ be an ordinal above $\sup(X)$ which has uncountable cofinality and which is a regular cardinal in $L[X]$ then λ is one of the canonical indescernibles for X.

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Since in $X \in \mathcal{C}(Q^{cf}_\omega)$ then we can find in $\mathcal{C}(Q^{cf}_\omega)$ arbitrarily long sequences of λ's satisfying the conditions in the lemma, so we can find enough indescernibles for X to define $X^\sharp$.

Some more closure of $\mathcal{C}(Q_\omega^{cf})$

$\mathcal{C}(Q_\omega^{cf})$ is closed under a large variety of definable operations for instance:

Theorem

- *If $X \in \mathcal{C}(Q_\omega^{cf})$ is a set of ordinals then the Dodd-Jensen core models over X is included in $\mathcal{C}(Q_\omega^{cf})$.*
- *If there is a measurable cardinal then for every set of ordinals $X \in \mathcal{C}(Q_\omega^{cf})$ there is an inner model in $\mathcal{C}(Q_\omega^{cf})$ containing X and a measurable cardinal.*

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Robustness of $\mathcal{C}(Q^{cf}_{\omega})$

By forcing over L one can change $\mathcal{C}(Q^{cf}_{\omega})$. In particular make it violate the Continuum Hypothesis.

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- *The set of reals of $\mathcal{C}(Q^{cf}_{\omega})$ is not changed by set forcing.*
- *The theory of $\mathcal{C}(Q^{cf}_{\omega})$ is the same as the theory of $\mathcal{C}(Q^{cf}_{<\kappa})$ for every regular cardinal κ. (The quantifier $Q^{cf}_{<\kappa}xy\Phi(x,y)$ means "$\Phi(x,y)$ defines a linear order whose cofinality is less than κ".)*

Limiting the completeness of $\mathcal{C}(Q_\omega^{cf})$

Theorem

Suppose there is a Woodin cardinal and that $M_1^\sharp$ exists. ($M_1^\sharp$ is a countable canonical model for Woodin cardinal with its sharp.) then every real of $\mathcal{C}(Q_\omega^{cf})$ appears in $M_1^\sharp$.

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We know it can contain inner models for measurables . Since it contains the Dodd-Jensen Core Model , if our universe is the Dodd-Jensen core model then $V = \mathcal{C}(Q_\omega^{cf})$. Hence it can contain cardinals like Ramsey cardinals.

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Theorem

Assume $V = L_{\mu}$. Let M_{α} be the α's iterate of V by the (unique) normal measure on the unique measurable cardinal. Then $\mathcal{C}(Q^{cf}_{\omega}) = M_{\omega^2}[P]$ where P is a Prikry sequence of the unique measurable cardinal of M_{ω^2}. In particular $\mathcal{C}(Q^{cf}_{\omega})$ has no measurable cardinal.

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Theorem

Assume there is a supercompact cardinal κ then one of the following holds:

1. *$\mathcal{C}(Q^{aa})$ is a κ covering model for V. i.e. For every X, a set of ordinals of cardinality less than κ is covered by a set $Y \in \mathcal{C}(Q^{aa})$ such that $|Y| < \kappa$.*
2. *Every uncountable regular cardinal greater or equal κ is measurable in $\mathcal{C}(Q^{aa})$.*

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The first alternative seems rather unlikely.

Theorem

Assume the forcing axiom MM . Then every regular uncountable cardinal is measurable in $\mathcal{C}(Q^{aa})$

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This is very much in the spirit of the great logician we remembering today: Andrej Mostowski

Thank You!